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The supersymmetric-flavor problem for heavy first-two-generation scalars at next-to-leading order

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Abstract. We analyze in detail the constraints on SUSY-model parameters obtained from K–$\overline{K}$ mixing in the hypothesis of a splitted SUSY spectrum. FCNC contributions from gluino–squark–quark interactions are studied in the so-called mass-insertion approximation. We present boundaries on mass insertions and on SUSY mass scales. We improve on previous results by including the NLO-QCD corrections to the $\Delta S = 2$ effective Hamiltonian and the complete set of B parameters for the evaluation of hadronic matrix elements. A full set of magic numbers that can be used for further analyses of these models is also given. We find that the inclusion of NLO-QCD corrections and the B parameters change the results obtained at LO and in the vacuum insertion approximation by an amount of about 25–35%.

1 Introduction

It is well known that supersymmetry (SUSY) introduces many new sources of flavor-changing neutral currents (FCNC) which give strong constraints on the construction of extensions of the standard model (SM).

A common feature of these models is that FCNC effects are induced by SUSY-breaking parameters that mix different flavors. In the literature, several ideas have been proposed in order to suppress unwanted FCNC effects. For instance, in models where SUSY breaking is induced by gauge interactions [1], SUSY-breaking parameters are either flavor-blind or are dominated by the dilaton multiplet of string theory [2]. Alternatively, flavor symmetries are used to provide either a sufficient degeneracy between the first two generations of sfermions [3] or an alignment between quark and squark mass matrices [4].

Here we want to investigate the hypothesis that the average squark mass of the first two generations is much higher then the rest of the spectrum of (s)particles [5–8]. Throughout the paper we indicate the average mass of the heavy scalar squarks as $M_{\rm sq}$ and the typical mass scale of gauginos and of the other light sparticles as $m_{\tilde{g}}$. Small Yukawa couplings of the first two generations of scalars to Higgs doublets, together with masses of the rest of the supersymmetric spectrum close to the weak scale, allow a natural electroweak symmetry breaking (EWSB). This scenario has very interesting phenomenological signatures [8] and can be easily realized in string theory [9].

We consider gluino–squark-mediated FCNC contributions to ΔM_K and ϵ_K in the neutral K-system. The effect of the most general squark mass matrix for this class of models is studied. In some cases, further restrictions on the squark masses are required, and other contributions can be more important. In particular, chargino–squark–quark interactions should be considered also. We postpone a discussion that would include these effects to a subsequent work.

We work in the so-called mass-insertion approximation [10]. In this framework, one chooses a basis for fermion and sfermion states where all the couplings of these particles to neutral gauginos are flavor-diagonal and flavor-changing (FC) effects are shown by the nondiagonality of sfermion propagators. The pattern of flavor change, for the K system, is given by the ratio

$$(\delta^d_{ij})_{AB} = \frac{(m^{\tilde{d}}_{ij})^2_{AB}}{M^2_{\rm sq}} \ , \tag{1}$$

where $(m^{\tilde{d}}_{ij})^2_{AB}$ are the off-diagonal elements of the $\tilde{d}$-mass-squared matrix that mixes flavors i, j for both left- and right-handed scalars (A, B =left, right); see, e.g., [11]. The sfermion propagators are expanded as a series in terms of δs, and the contribution of the first term of this expansion is considered.

The supersymmetric flavor problem consists in building viable models in which FCNC are suppressed without requiring excessive fine tuning of the parameters.

In models with a split spectrum of sparticles, in which the average mass of the lightest ($m_{\tilde{g}}$) is in the electroweak or TeV region, two scenarios are possible:

1. For reasonable values of $M_{\rm sq}$, the suppression of FCNC requires small δs values. Thus, by fixing $M_{\rm sq}$, one can find constraints on δs; see [11,12], and for a very recent NLO analysis, [13];
2. For natural values of δs, say $\mathcal{O}(1)$ or the order of the Cabibbo angle, $\mathcal{O}(0.22)$, one finds that the only way to

get rid of unwanted FCNC effects is to have squarks of the first two generations that are heavy enough. Thus, by fixing the δs, one can find constraints on the minimal values for M_{sq}. Large values of M_{sq}, however, induce large values for the GUT masses of the third generation of squarks via Renormalization-group equations (RGE). Consequently, there can be fine-tuning problems for the Z-boson mass. We study this issue in Sect. 4. This point of view is adopted in [16,17].

In the past, several phenomenological analyses have been carried out which rely on some approximations. For instance, [16] does not include QCD radiative corrections, and makes use of vacuum insertion approximation (VIA) for the evaluation of hadronic matrix elements. Leading-order QCD corrections to the evolution of Wilson coefficients are considered in the papers of [12,17]; these authors find that QCD corrections are extremely important. For example, in [17], the lower bound on the heavy squark mass is increased by approximately a factor of 3.

In this work we discuss both of the cited scenarios and improve on previous analyses, including the next-to-leading order (NLO) QCD corrections to the most general $\Delta F = 2$ effective Hamiltonian [18], and the lattice calculation of all the B parameters appearing in the K–$\bar{\mathrm{K}}$-mixing matrix elements that have been recently computed [20]. We find this very interesting for several reasons. First of all, we find that the inclusion of these effects leads to sizeable deviation from the previous computations. The results obtained using only LO-QCD corrections and VIA are corrected by about 25–35%. Furthermore, the uncertainty in the final result, due to its dependence on the scale at which hadronic matrix elements and quark masses are evaluated, is much reduced.

Predictions for any model can be tested using the magic numbers we provide. These numbers allow us to obtain the coefficient functions at any low-energy scale, once the matching conditions are given at a higher-energy scale. The magic numbers will be useful, e.g., when a complete NLO analysis of SUSY contributions to $\Delta F = 2$ processes (which should include also chargino-exchange effects) will be implemented in the future.

A complete NLO calculation should include also the $\mathcal{O}(\alpha_s)$ corrections to the Wilson coefficients at the scale of the SUSY masses running in the loops. So far, we are missing this piece of information for gluino–squark contributions[1]. We can argue the smallness of these corrections from the smallness of α_s at such scales. This uncertainty can be removed only by a direct computation.

The paper is organized as follows. In Sect. 2 we introduce the formalism concerning the operator basis, the Wilson coefficients and the RGE. In Sect. 3, constraints on δs are derived. The problem of consistency of the squark spectrum for given entries of δs' is considered in Sect. 4. Finally our conclusions are given in Sect. 5, and all the magic numbers are given in the appendix.

[1] The matching conditions for charged-Higgs and chargino contributions have been recently computed [22].

2 Effective Hamiltonian and hadronic matrix elements

In this section we describe the framework in which the basic calculations have been performed. We follow the discussion of [12] in the case $M_{\mathrm{sq}} \gg m_{\tilde{g}}$. Throughout the paper (unless otherwise explicitly specified), we assume that the average mass of gluinos and of the squarks of the third generation are of the same order of magnitude.

The three steps needed to use the operator product expansion (OPE) (matching of the effective theory, perturbative evolution of the coefficients, and evaluation of hadronic matrix elements) are treated in detail in the following subsections.

2.1 Operator basis and matching of the effective theory

In order to apply the OPE, one has to calculate the coefficients and operators of the effective theory. One first integrates out the heavy scalars of the first two generations at the scale M_{sq}. This step produces $\Delta S = 1$ (of the form $\bar{d}\tilde{g}\bar{\tilde{g}}s$) as well as $\Delta S = 2$ operators, at the same order, $1/M_{\mathrm{sq}}^2$. When gluinos are also integrated out at $m_{\tilde{g}}$, $\Delta S = 1$ operators generate $\Delta S = 2$ contributions that are proportional to $m_{\tilde{g}}^2/M_{\mathrm{sq}}^4$, and so can be neglected.

The final basis of operators is:

$$\begin{aligned}
Q_1 &= \bar{d}^\alpha \gamma_\mu (1-\gamma_5) s^\alpha \; \bar{d}^\beta \gamma_\mu (1-\gamma_5) s^\beta \,, \\
Q_2 &= \bar{d}^\alpha (1-\gamma_5) s^\alpha \; \bar{d}^\beta (1-\gamma_5) s^\beta \,, \\
Q_3 &= \bar{d}^\alpha (1-\gamma_5) s^\beta \; \bar{d}^\beta (1-\gamma_5) s^\alpha \,, \\
Q_4 &= \bar{d}^\alpha (1-\gamma_5) s^\alpha \; \bar{d}^\beta (1+\gamma_5) s^\beta \,, \\
Q_5 &= \bar{d}^\alpha (1-\gamma_5) s^\beta \; \bar{d}^\beta (1+\gamma_5) s^\alpha \,,
\end{aligned} \tag{2}$$

together with operators $\tilde{Q}_{1,2,3}$ which can be obtained from $Q_{1,2,3}$ by the exchange $(1-\gamma_5) \leftrightarrow (1+\gamma_5)$.

The Wilson coefficients at the matching scale M_{sq} are (see, e.g., [11,12]):

$$\begin{aligned}
C_1 &= -\frac{\alpha_s^2}{216 M_{\mathrm{sq}}^2} \left(24 x f_6(x) + 66 \tilde{f}_6(x)\right) (\delta_{12}^d)^2_{\mathrm{LL}}, \\
C_2 &= -\frac{\alpha_s^2}{216 M_{\mathrm{sq}}^2} 204 f_6(x) (\delta_{12}^d)^2_{\mathrm{RL}}, \\
C_3 &= \frac{\alpha_s^2}{216 M_{\mathrm{sq}}^2} 36 x f_6(x) (\delta_{12}^d)^2_{\mathrm{RL}}, \\
C_4 &= -\frac{\alpha_s^2}{216 M_{\mathrm{sq}}^2} \left[\left(504 x f_6(x) - 72 \tilde{f}_6(x)\right) (\delta_{12}^d)_{\mathrm{LL}} (\delta_{12}^d)_{\mathrm{RR}} \right. \\
&\quad \left. -132 \tilde{f}_6(x) (\delta_{12}^d)_{\mathrm{LR}} (\delta_{12}^d)_{\mathrm{RL}}\right], \\
C_5 &= -\frac{\alpha_s^2}{216 M_{\mathrm{sq}}^2} \left[\left(24 x f_6(x) + 120 \tilde{f}_6(x)\right) (\delta_{12}^d)_{\mathrm{LL}} (\delta_{12}^d)_{\mathrm{RR}} \right. \\
&\quad \left. -180 \tilde{f}_6(x) (\delta_{12}^d)_{\mathrm{LR}} (\delta_{12}^d)_{\mathrm{RL}}\right],
\end{aligned} \tag{3}$$

where $x = (m_{\tilde{g}}/M_{\rm sq})^2$, and

$$f_6(x) = \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5},$$
$$\tilde{f}_6(x) = \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}. \quad (4)$$

The coefficients for the operators $\tilde{Q}_{1,2,3}$ are the same as those of $Q_{1,2,3}$ with the replacement $\mathrm{L} \leftrightarrow \mathrm{R}$. The authors of [12,16,17] use the matching coefficients directly in the limit $x \to 0$. However, we have contemplated also the extreme case of $m_{\tilde{g}} \sim M_{\rm sq}/2$, so that we keep the whole expression. Of course, the value of the coefficients is the same as that of [12,16,17] in cases where $x \ll 1$.

As we have said, NLO corrections to these coefficients have not been computed yet. We assume they are negligible, in view of the smallness of $\alpha_s(M_{\rm sq})$ and of the fact that similar corrections turn out to be rather small in the SM and the two-Higgs doublet model, and for the chargino contribution in the constrained MSSM [22]. Our effective Hamiltonian is so affected by a residual renormalization-scheme dependence because of the missing piece of $\mathcal{O}(\alpha_s(M_{\rm sq}))$ in the matching.

2.2 Evolution of Wilson coefficients and running of α_s

In order to evolve the Wilson coefficients between $M_{\rm sq}$ and the scale at which hadronic matrix elements are evaluated ($\mu = 2$GeV), one has to account for the presence of all particles whose mass is intermediate between the two scales, both in the β function of α_s and in the anomalous dimension matrix (ADM) of the operators. Regarding the former, one has [25]:

$$\beta(\alpha_s) = -\beta_0\alpha_s^2 - \beta_1\alpha_s^3 + \mathcal{O}(\alpha_s^4), \quad (5)$$
$$\beta_0 = \frac{1}{3}\left(11N_c - 2n_f - 2N_c n_{\tilde{g}} - \frac{1}{2}n_{\tilde{q}}\right), \quad (6)$$
$$\beta_1 = \frac{1}{3}\left(34N_c^2 - \frac{13N_c^2-3}{N_c}n_f - 16N_c^2 n_{\tilde{g}} - \frac{4N_c^2-3}{2N_c}n_{\tilde{q}} + 3\,\frac{3N_c^2-1}{2N_c}n_{\tilde{q}}n_{\tilde{g}}\right), \quad (7)$$

where $N_c = 3$ for the color SU(3), and n_f is the number of fermion flavors. The terms proportional to $n_{\tilde{g}}$ and $n_{\tilde{q}}$ represent, respectively, the gluino and light scalar contributions. One has $n_{\tilde{g}}$=1 and $n_{\tilde{q}}$=4 evolving between $M_{\rm sq}$ and $m_{\tilde{g}}$ and $n_{\tilde{g}} = n_{\tilde{q}}$=0 evolving from $m_{\tilde{g}}$ to a lower mass scale.

In [18], the ADM of the operators was computed at NLO. In that paper, since all SUSY particles are taken to be heavy, only loops with fermions and gluons are considered. This result must be modified to take into account that, from $M_{\rm sq}$ to $m_{\tilde{g}}$, the squarks of the third generation, as well as gluinos, can also run in the loops. As a matter of fact, for the K system, light third-generation squarks and gluinos can enter two-loop ADMs only via the renormalization of the gluon propagator. An explicit calculation shows that the required modification consists in considering the ADM of [18] as a function of $n_f + N_c\; n_{\tilde{g}} + n_{\tilde{q}}/4$ when one evolves between the heavy squark and gluino mass scales, and as a function of n_f below the latter scale. This substitution is no more true if the squarks of the first two generations are light too.

The value of the Wilson coefficients at the hadronic scale $\mu = 2$ GeV, where matrix elements are computed, can then be easily calculated. Following [18], one evolves between two scales according to:

$$\mathbf{C}(\mu) = \hat{N}[\mu]\; \hat{U}[\mu, M]\; \hat{N}^{-1}[M]\; \mathbf{C}(M),$$
$$\hat{N}[\mu] = \hat{1} + \frac{\alpha_s(\mu)}{4\pi}\; \hat{J}(\mu),$$
$$\hat{U}[\mu, M] = \left[\frac{\alpha_s(M)}{\alpha_s(\mu)}\right]^{\hat{\gamma}^{(0)T}/(2\beta_0)}, \quad (8)$$

where $\hat{\gamma}^{(0)}$ is the LO-ADM, and $\mathbf{C}(\mu)$ are the Wilson coefficients arranged in a column vector. This formula is correct up to the NLO. $\hat{U}[\mu, M]$ gives the LO evolution already computed in [12] while $\hat{J}$ gives the NLO corrections calculated in [18]. $\hat{J}$ depends on both the number of active particles at the scale μ and the renormalization scheme used for its computation. We have used $\hat{J}$ in the same scheme used for the lattice calculation of hadronic matrix elements, the Landau–RI scheme (LRI). In this way the renormalization scheme dependence of the final result, at the scale at which hadronic matrix elements are evaluated, cancels out at this perturbative order. As already stressed, for a complete scheme independence of our result, one should include also the NLO corrections of the Wilson coefficients at the high matching scale.

We provide here the full set of Wilson coefficients at μ=2 GeV as functions of $M_{\rm sq}$ and $m_{\tilde{g}}$ (the so-called magic numbers). We find

$$C_i(\mu) = \sum_{r,j=1}^{5}\left[b_{ij}^{(r)} + \frac{\alpha_s(m_{\tilde{g}})}{4\pi}c_{ij}^{(r)}\right]\; \alpha_s^{a_r}(m_{\tilde{g}})\; C_j(m_{\tilde{g}}),$$
$$C_i(m_{\tilde{g}}) = \sum_{r,j=1}^{5}\left[d_{ij}^{(r)} + \frac{\alpha_s(m_{\tilde{g}})}{4\pi}e_{ij}^{(r)} + \frac{\alpha_s(M_{\rm sq})}{4\pi}f_{ij}^{(r)}\right] \times \left(\frac{\alpha_s(M_{\rm sq})}{a_s(m_{\tilde{g}})}\right)^{a'_r} C_j(M_{\rm sq}). \quad (9)$$

The complete expression of $a_r, a'_r, b_{ij}^{(r)}, \ldots$, is given in the appendix. Equation (9) is useful for testing predictions for any model, once the two scales are fixed. The magic numbers for the evolution of $\tilde{C}_{1-3}$ are the same as the ones for the evolution of C_{1-3}. Equation (9) and the formulas of the appendix can be used with B parameters evaluated at $\mu = 2$ GeV (see (12)), in order to determine the contribution to ΔM_K and ϵ_K at NLO in QCD for any model of new physics in which the new contributions with respect to the SM originate from the extra-heavy particles. It is sufficient to compute the values of the coefficients at the matching scales $M_{\rm sq}$ and $m_{\tilde{g}}$ and put them in (9).

2.3 Hadronic matrix elements

The hadronic matrix elements of the operators of (2) in the VIA are:

$$\begin{aligned}
\langle K^0|Q_1|\bar{K}^0\rangle_{\rm VIA} &= \frac{1}{3}M_K f_K^2,\\
\langle K^0|Q_2|\bar{K}^0\rangle_{\rm VIA} &= -\frac{5}{24}\left(\frac{M_K}{m_s+m_d}\right)^2 M_K f_K^2,\\
\langle K^0|Q_3|\bar{K}^0\rangle_{\rm VIA} &= \frac{1}{24}\left(\frac{M_K}{m_s+m_d}\right)^2 M_K f_K^2,\\
\langle K^0|Q_4|\bar{K}^0\rangle_{\rm VIA} &= \left[\frac{1}{24}+\frac{1}{4}\left(\frac{M_K}{m_s+m_d}\right)^2\right] M_K f_K^2,\\
\langle K^0|Q_5|\bar{K}^0\rangle_{\rm VIA} &= \left[\frac{1}{8}+\frac{1}{12}\left(\frac{M_K}{m_s+m_d}\right)^2\right] M_K f_K^2, \quad (10)
\end{aligned}$$

where M_K is the mass of the K meson and m_s, m_d are the masses of the s and d quarks, respectively. An analogous definition holds for $\tilde{Q}_{1,2,3}$.

Hadronic matrix elements can be evaluated nonperturbatively introducing B parameters, defined as follows:

$$\begin{aligned}
\langle K^0|Q_1(\mu)|\bar{K}^0\rangle &= \frac{1}{3}M_K f_K^2 B_1(\mu),\\
\langle K^0|Q_2(\mu)|\bar{K}^0\rangle &= -\frac{5}{24}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2\\
&\quad \times M_K f_K^2 B_2(\mu),\\
\langle K^0|Q_3(\mu)|\bar{K}^0\rangle &= \frac{1}{24}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_3(\mu),\\
\langle K^0|Q_4(\mu)|\bar{K}^0\rangle &= \frac{1}{4}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_4(\mu),\\
\langle K^0|Q_5(\mu)|\bar{K}^0\rangle &= \frac{1}{12}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2\\
&\quad \times M_K f_K^2 B_5(\mu), \quad (11)
\end{aligned}$$

where $Q_i(\mu)$ are the operators renormalized at the scale μ. The B parameters for $\tilde{Q}_{1,2,3}(\mu)$ are the same as those of $Q_{1,2,3}(\mu)$.

In the computation of B_i for the operators 2–5, smaller contributions of higher order in chiral expansion, coming from the axial current, have been neglected. A detailed explanation of the reasons for this approximation can be found in [20]. The definition of B parameters in (11) takes explicitly into account this approximation, and when it is used, the low-scale (μ) dependence of the final result is explicitly canceled in the product of coefficient functions and hadronic matrix elements.

The B parameter of the first operator, usually referred to as B_K, has been extensively studied on the lattice and used in many phenomenological applications (see, e.g., [23, 24]). We have considered its world average [23]. The other B_i have been taken from [20] (for another determination of these B_i, calculated with perturbative renormalization, see [21]).

Table 1. Constants used for phenomenological analysis

Constants	Values
$\alpha_{em}(M_Z)$	1/127.88
$\alpha_s(M_Z)$	0.119
M_K	497.67 MeV
f_K	159.8 MeV
$m_d(2\ \text{GeV})$	7 MeV
$m_s(2\ \text{GeV})$	125 MeV
m_c	1.3 GeV
m_b	4.3 GeV
m_t	175 GeV
$\sin^2\theta_W(M_Z)$	0.23124

All the B parameters are evaluated at a scale of 2 GeV in the LRI renormalization scheme:

$$\begin{aligned}
B_1\,(\mu = 2\ \text{GeV}) &= 0.60 \pm 0.06,\\
B_2\,(\mu = 2\ \text{GeV}) &= 0.66 \pm 0.04,\\
B_3\,(\mu = 2\ \text{GeV}) &= 1.05 \pm 0.12,\\
B_4\,(\mu = 2\ \text{GeV}) &= 1.03 \pm 0.06,\\
B_5\,(\mu = 2\ \text{GeV}) &= 0.73 \pm 0.10. \quad (12)
\end{aligned}$$

In the literature to date, all phenomenological analyses on this subject have used the VIA and have computed Wilson coefficients and quark masses at a scale varying from 0.5–1 GeV. We will see that this represents in some cases quite a rough approximation.

Finally, we give in Table 1 all the numerical values of the physical constants we have considered. All coupling constants and $\sin^2\theta_W(M_Z)$ are meant in the $\overline{MS}$ scheme [26].

3 Constraints on δs

We are ready to provide a set of constraints on SUSY variables coming from the $\mathrm{K_L}$– $\mathrm{K_S}$ mass difference, ΔM_K, and the CP violating parameter ϵ_K, defined as

$$\begin{aligned}
\Delta M_K &= 2\mathrm{Re}\langle K^0|H_{\rm eff}|\bar{K}^0\rangle,\\
\epsilon_K &= \frac{1}{\sqrt{2}\Delta M_K}\mathrm{Im}\langle K^0|H_{\rm eff}|\bar{K}^0\rangle. \quad (13)
\end{aligned}$$

The parameter space is composed of two real and four complex entries, that is $M_{\rm sq}$ and $m_{\tilde{g}}$, and $(\delta^d_{12})_{\rm LL}$, $(\delta^d_{12})_{\rm LR}$, $(\delta^d_{12})_{\rm RL}$, and $(\delta^d_{12})_{\rm RR}$, respectively.

Neglecting interference among different SUSY contributions, we give upper bounds on δs, at fixed values of $M_{\rm sq}$ and $m_{\tilde{g}}$, with the condition $M_{\rm sq} > m_{\tilde{g}}$. In this way one gets a set of constraints for each individual δs. Indeed, since we are interested in model-independent constraints, it is meaningful to study the interference of cancellation effects only in specific models.

The physical condition used to get the bounds on δs is that the SUSY contribution (proportional to each single δs) plus the SM contribution to ΔM_K and ϵ_K do not exceed the experimental value of these quantities. As far as the SM contribution to ΔM_K is concerned, we assume that the values of the CKM elements V_{cd} and V_{cs} are unaffected by SUSY. This implies the (very reasonable) hypothesis that SUSY does not significantly correct tree-level weak decays. The value of the SM contribution to ϵ_K instead depends on the phase of the CKM matrix. This phase can be largely affected by unknown SUSY corrections, and can be treated as a free parameter. We put the CKM phase at zero so that the experimental value of ϵ_K is completely determined by SUSY. Finally, to be even more conservative, we subtract one standard deviation from the values of the B parameters.

The final results are shown in Tables 2–7 for gluino masses of 250, 500, and 1000 GeV. We consider the heavy squark masses expected in some common models (see, e.g., [5–8]).

The constraints that come from the four possible insertions of δs are presented: in the first and second rows, only terms proportional to $(\delta^d_{12})_{\rm LL}$ and $(\delta^d_{12})_{\rm LR}$, respectively, are considered; in the last two rows, the contributions of operators with opposite chirality, RR and RL, are also evaluated, by assuming $(\delta^d_{12})_{\rm LR} = (\delta^d_{12})_{\rm RL}$ and $(\delta^d_{12})_{\rm LL} = (\delta^d_{12})_{\rm RR}$.

In each column of the table, we show the bounds on δs in the various approximations that one can use for their determination: without QCD correction and in VIA, with LO-QCD corrections and in VIA, with LO-QCD corrections and with lattice B parameters, and, eventually, with NLO-QCD corrections and lattice B parameters. Comparing the values of our constraints at LO-VIA with those found from the authors of [12], we find some differences. The reason is twofold. On the one hand, they do not consider the SM contribution to ΔM_K; on the other, they evaluate the hadronic matrix elements at a scale $\tilde{\mu}$, such that $\alpha_s(\tilde{\mu}) = 1$. This latter choice may be questionable, because at this scale strong interactions break perturbation theory.

The combination of B parameters and NLO-QCD corrections changes the LO-VIA results by about 25–35%. As expected [12], the tightest constraints are for the cases $(\delta^d_{12})_{\rm LL} = (\delta^d_{12})_{\rm RR}$ and $(\delta^d_{12})_{\rm LR} = (\delta^d_{12})_{\rm RL}$. In these cases, the coefficients proportional to $(\delta^d_{12})_{\rm LL}(\delta^d_{12})_{\rm RR}$, $(\delta^d_{12})_{\rm LR}$ $(\delta^d_{12})_{\rm RL}$ dominate the others.

We have checked that the uncertainties of the results, due to higher perturbative orders, are sizeable: as high as 10% in some cases.

4 Constraints on squarks spectrum

In this section, following the discussion of [16], we provide a different kind of constraint.

For fixed values of δs and of the average light sparticle mass, $m_{\tilde{g}}$, it is possible to calculate the minimum value of $M_{\rm sq}$ necessary to suppress the FCNC at an experimentally acceptable level. Here we give constraints on

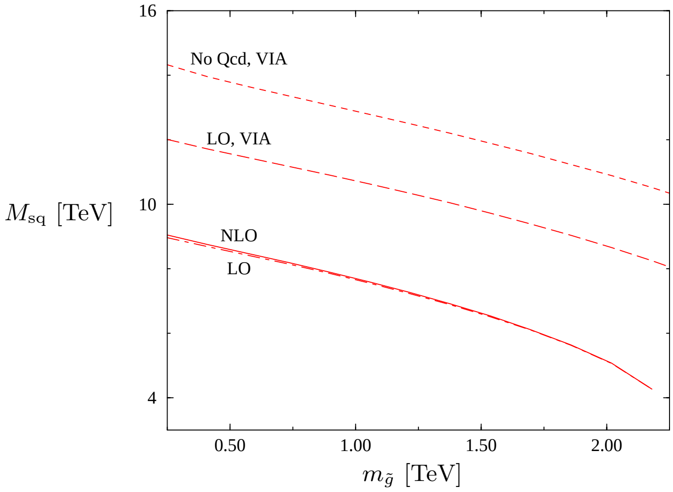

Fig. 1. Lower bounds on $M_{\rm sq}$ from ΔM_K with various approximations for case I with $\mathcal{K} = 0.22$. In this case, the larger corrections to LO-VIA come from the B parameters

$M_{\rm sq}$ and discuss their consistency. Using renormalization-group equations, one finds that a too-large $M_{\rm sq}$ can drive the average mass of the third generation of sfermions, $m_{\tilde{f}}$, to zero or negative values at the TeV scale. To circumvent this problem, a minimum value for $m_{\tilde{f}}$ $(\mu_{\rm GUT})$ at the GUT scale has to be chosen. If $m_{\tilde{f}}(\mu_{\rm GUT})$ is too high (say, more then 3–4 TeV), however, a too- large fine-tuning of the SUSY parameters is required in order to account for the observed mass of the Z boson, and severe naturalness problems arise [14,15]. This problem is studied in [16,17].

One obtains constraints about the consistency of models with a splitted mass spectrum following three steps:

– determining the *minimum* value of $M_{\rm sq}$ necessary to suppress FCNC (this is discussed in Sect. 4.1);
– computing the *maximum* value of $M_{\rm sq}$ allowed by positiveness of light scalar masses and fine-tuning (for more about this, see Sect. 4.2);
– combining the previous two results one can determine regions of allowable values of $M_{\rm sq}$ that satisfy both the requests of the previous points (we comment about this in Sect. 4.3).

4.1 Minimum values for heavy squark mass

In order to obtain constraints on $M_{\rm sq}$ one has to specify a value for δs. We consider the cases

$$\begin{array}{ccccc} & (\delta^d_{12})_{\rm LL} & (\delta^d_{12})_{\rm LR} & (\delta^d_{12})_{\rm RL} & (\delta^d_{12})_{\rm RR} \\ \mathrm{I} & \mathcal{K} & 0 & 0 & 0 \\ \mathrm{II} & 0 & \mathcal{K} & 0 & 0 \\ \mathrm{III} & \mathcal{K} & 0 & 0 & \mathcal{K} \\ \mathrm{IV} & 0 & \mathcal{K} & \mathcal{K} & 0 \end{array} \qquad (14)$$

where $\mathcal{K}$ can take the values (1, 0.22, 0.05). We have chosen these entries to leave aside possible accidental cancel-

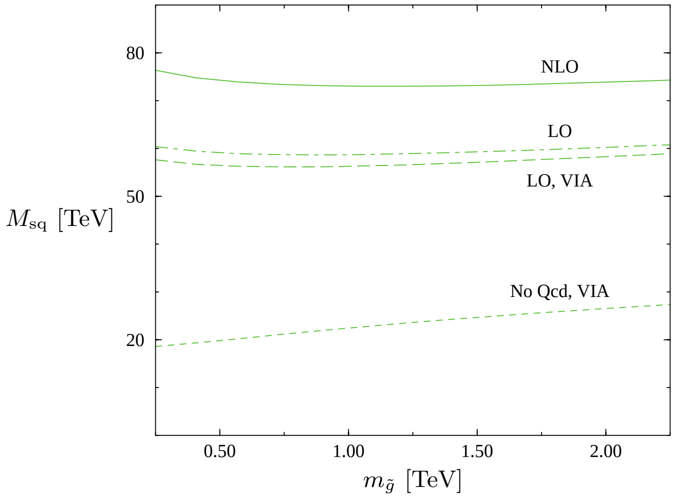

Fig. 2. Lower bounds on $M_{\rm sq}$ from ΔM_K with various approximations for case III with $\mathcal{K} = 0.22$. In this case, the larger corrections to LO-VIA come from NLO perturbative corrections

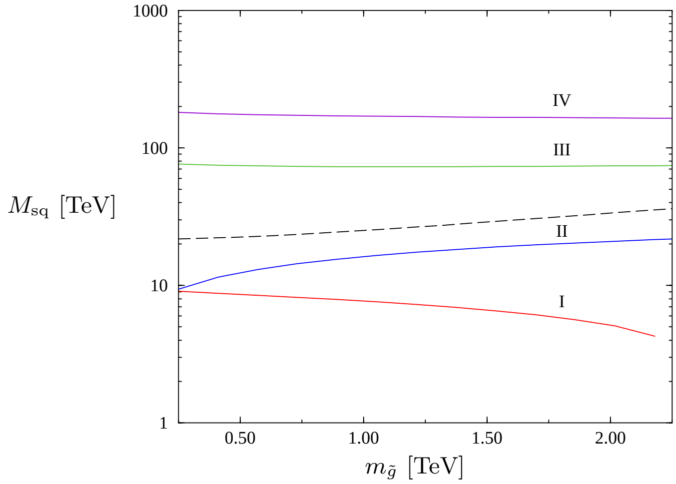

Fig. 4. The same as in Fig. 3, with $\mathcal{K}$=0.22. Cases I and II are now compatible with fine-tuning requirements

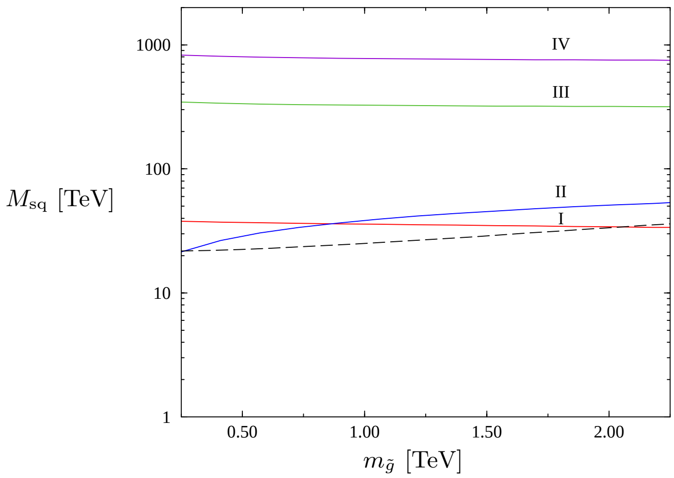

Fig. 3. The full (colored) lines give the lower bounds on $M_{\rm sq}$ necessary to suppress FCNC, with $\mathcal{K}$=1 for the various cases. An upper bound on $M_{\rm sq}$ is derived in order to satisfy fine-tuning requirements; it is shown by the dashed line. The two kinds of constraints are not compatible in this case

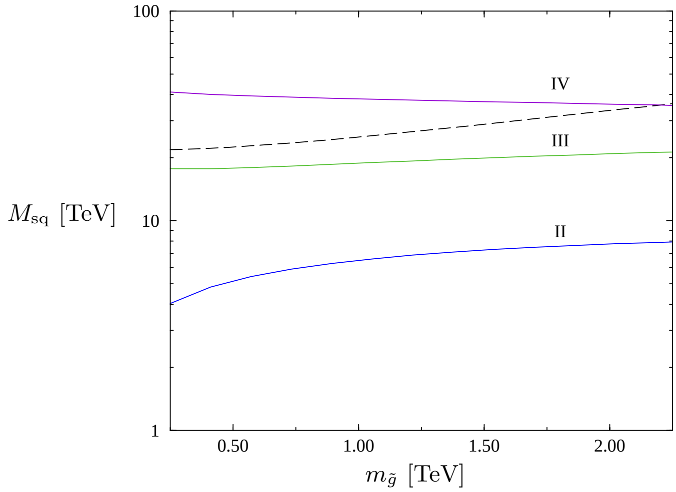

Fig. 5. The same as in Fig. 3, with $\mathcal{K}$=0.05. Case I is not drawn since no lower bound on $M_{\rm sq}$ can be obtained in this case. Cases II and III are now compatible with fine-tuning requirements

lations. The cases in which $\mathcal{K}$=1, are, of course, extreme cases: One may wonder about the consistency of mass-insertion approximation, as the neglected terms are of order $\mathcal{O}(1)$. However, these cases have already been studied in the literature, (see, e.g., [16,17]), and we report them for completeness. The results so obtained just give an estimate of the mass scales that are involved, and can be trusted if other corrections do not provide accidental cancellations. This can be checked only by a direct calculation. The assumptions made for the SM contribution and the B parameters are the same as in Sect. 3.

In order to monitor the effect of the different corrections on the final result, we show in Figs. 1, 2 the lower bound obtained for cases I and III with $\mathcal{K} = 0.22$ (the other cases give similar results). As we see, B parameters and NLO-QCD corrections play a significant rôle in the final computation and the correction they provide with respect to the LO-VIA results are of the order of 25–35%. In particular, in case I (Fig. 1), B parameters provide the most important corrections with respect to LO-VIA results. In case III (Fig. 2), instead, corrections to LO-VIA results are dominated by the NLO-QCD perturbative contributions.

The case $(\delta^d_{12})_{\rm LL} = (\delta^d_{12})_{\rm RR}$ is also considered in [16, 17]. The differences, at LO and without B-parameters, between our result and the ones of [16,17] come from (i) our inclusion of the SM contribution; (ii) the value of the strange-quark mass; and (iii) the scale at which hadronic matrix elements are evaluated. We agree with [16,17] for the same choice of parameters.

Notice that if the imaginary parts of δs are of the same order as their real parts, much stronger constraints arise from ϵ_K than from ΔM_K (namely by a factor ~ 7.7). To be conservative, we consider in this section only constraints coming from the real parts of δs.

The final results are shown by the (colored) continuous lines in Figs. 3, 4, 5. The minimum value of $M_{\rm sq}$ depends strongly on both $\mathcal{K}$ and on the case one considers (I, II, III, or IV; see (14)). Notice that $(\delta^d_{12})_{\rm LR}$, $(\delta^d_{12})_{\rm RL}$, (which enter into cases II and IV), are "naturally" small in the MSSM. However, since we would like to do a model-independent analysis, we have made no particular assumptions about them. In all graphs, the strongest constraints come from the case $(\delta^d_{12})_{\rm LR} = (\delta^d_{12})_{\rm RL} \neq 0$. Much lower constraints are generally obtained in cases I and II. In Fig. 5, case I has not been drawn, since no constraint can be derived.

4.2 RGE for the masses of the third generation of scalars

It is well known that large values of $M_{\rm sq}$ can drive the mass of the third generation of scalars to negative values via RGE. Let us consider the two-loop RGEs for the mass $m_{\tilde f}$ of the third generation of scalars, $\tilde f$. In the $\overline{DR}\,'$ scheme (see, e.g., [27]), with two generations of heavy scalars, one has

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_{\tilde f}(\mu) = -\frac{8}{4\pi}\sum_i \alpha_i(\mu)\; C_i^{\tilde f}(m_G^2)_i(\mu) + \frac{32}{(4\pi)^2}\sum_i \alpha_i^2(\mu)\; C_i^{\tilde f} M_{\rm sq}^2, \qquad (15)$$

where $C_i^{\tilde f}$ is the Casimir factor for $\tilde f$ in the SU(5) normalization, the sums are over the gauge groups SU(3), SU(2), and U(1), and m_G denotes the gaugino masses. In (15), Yukawa couplings are neglected: These couplings drive the light masses to even lower values and so, in this respect, our choice is a conservative one. Moreover, the introduction of Yukawa interactions requires further assumptions on SUSY parameters (see, e.g., [17]) that we do not discuss in this paper.

The solution of (15) between a grand unification theory (GUT) scale $\mu_{\rm GUT} \sim 2\times 10^{16}$ GeV, and $\mu \sim 1$ TeV can be easily written as

$$\begin{aligned} m^2_{\tilde f}(\mu) = m^2_{\tilde f}(\mu_{\rm GUT}) &- \sum_i \frac{16}{4\pi\beta_i^0}\Big[\alpha_i(\mu_{\rm GUT}) - \alpha_i(M_{\rm sq})\Big]C_i^{\tilde f}\; M^2_{\rm sq} \\ &+ \sum_i \frac{2}{\beta_i^0}\Big[m_G^2(\mu_{\rm GUT}) - (m_G^2)_i(M_{\rm sq})\Big]C_i^{\tilde f} \\ &+ \sum_i \frac{2}{\beta_i^0}\Big[(m_G)_i^2(M_{\rm sq}) - (m_G^2)_i(\mu)\Big]C_i^{\tilde f}, \end{aligned} \qquad (16)$$

where β_i^0 are the β-function LO coefficients of the ith gauge coupling. In (16), we have considered a common gaugino mass m_G at the GUT scale; as for the couplings, we have evolved them starting backward from $\mu = M_Z$. Note that in (16), the contribution of the heavy scalars has been decoupled at $M_{\rm sq}$.

Equation (16) can be used in order to derive consistency constraints on the values of $M_{\rm sq}$ and $m_G^2(\mu_{\rm GUT})$ once the values of $m^2_{\tilde f}(\mu)$ and of $m^2_{\tilde f}(\mu_{\rm GUT})$ are fixed. The latter can be determined according to the following requirements. First, $m^2_{\tilde f}(\mu)$ must be at least positive, such as to leave color and electric symmetries unbroken. The value of $m^2_{\tilde f}(\mu_{\rm GUT})$ determines the amount of fine-tuning necessary in order to achieve the electroweak symmetry breaking. Following [14], the necessary fine-tuning scales approximately as $10\% \times (0.3\ {\rm TeV}/m_{\tilde Q_3}(\mu_{\rm GUT}))^2$ for the squark doublet of the third generation $\tilde Q_3$. We have calculated the constraints on $M_{\rm sq}$ and $m_G^2(\mu_{\rm GUT})$ coming from (16) in the case $\tilde f = \tilde Q_3$, choosing for $m^2_{\tilde Q_3}(\mu_{\rm GUT})$ the value of $(3.5\ {\rm TeV})^2$. The latter choice corresponds to a fine-tuning of more than 0.1%.

At fixed values of $m^2_{\tilde Q_3}(\mu)$ and $m^2_{\tilde Q_3}(\mu_{\rm GUT})$ (which depend on $M_{\rm sq}$ and m_G), one can plot the upper value of $M_{\rm sq}$ as function of m_G. The result is the (black) dashed line of Figs. 3, 4, 5. One finds that $M_{\rm sq}$ cannot be much larger than about 25 TeV. Of course, this is just an estimate of this limiting value. The inclusion of Yukawa couplings, of more severe fine-tuning requirements, and of other effects can only lower this limit.

4.3 Final remarks

In Figs. 3, 4, 5, we combine the constraints derived in the two previous subsections. These figures (together with Tables 2–7) suggest that also models with a split mass spectrum need further assumptions to be phenomenologically viable, i.e., one has to introduce flavor symmetry or dynamical generation of degenerate scalar masses [16].

Without these further hypotheses, most of the cases that we have considered face fine-tuning problems. In particular, values of $\mathcal{K} \sim \mathcal{O}(1)$ are hardly acceptable. Although $\mathcal{K} \sim \mathcal{O}(0.22)$ and $\mathcal{K} \sim \mathcal{O}(0.05)$ have better chances, they must be treated carefully.

5 Conclusions

In this work, we analyze in detail the constraints on SUSY-model parameters coming from K–$\overline{\rm K}$ oscillations in the hypothesis of a split SUSY spectrum. FCNC contributions coming from gluino–squark–quark interactions, working in the so-called mass-insertion approximation, have been considered. We provide boundaries on mass insertions and on SUSY mass scales, and we discus their consistency. Previous results, including NLO-QCD corrections to the $\Delta S = 2$ effective Hamiltonian, and B parameters for the evaluation of hadronic matrix elements, have been improved. A full set of magic numbers is provided, that can be used for further analyses.

Table 2. Limits on $\mathrm{Re}(\delta^d_{12})_{AB}$ from ΔM_K with gaugino masses of 250 GeV

M_{sq} [TeV]	No-QCD, VIA	LO-VIA	LO, B_i	NLO, B_i
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}\rvert}$		
2	3.1×10^{-2}	3.6×10^{-2}	4.9×10^{-2}	4.9×10^{-2}
5	7.5×10^{-2}	8.8×10^{-2}	0.12	0.12
10	0.15	0.18	0.25	0.24
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	2.1×10^{-2}	1.4×10^{-2}	1.8×10^{-2}	1.6×10^{-2}
5	9.8×10^{-2}	6.5×10^{-2}	8.2×10^{-2}	7.2×10^{-2}
10	0.34	0.22	0.28	0.25
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}=\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	6.6×10^{-3}	3.5×10^{-3}	3.5×10^{-3}	2.8×10^{-3}
5	1.5×10^{-2}	7.7×10^{-3}	7.9×10^{-3}	6.4×10^{-3}
10	3.0×10^{-2}	1.5×10^{-2}	1.5×10^{-2}	1.2×10^{-2}
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}=\mathrm{Re}(\delta^d_{12})^2_{\mathrm{RR}}\rvert}$		
2	1.1×10^{-2}	5.2×10^{-3}	5.1×10^{-3}	4.1×10^{-3}
5	4.1×10^{-2}	1.6×10^{-2}	1.6×10^{-2}	1.3×10^{-2}
10	0.10	3.6×10^{-2}	3.4×10^{-2}	2.7×10^{-2}

Table 3. Limits on $\mathrm{Re}(\delta^d_{12})_{AB}$ from ΔM_K with gaugino masses of 500 GeV

M_{sq} [TeV]	No-QCD, VIA	LO-VIA	LO, B_i	NLO, B_i
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}\rvert}$		
2	3.8×10^{-2}	4.5×10^{-2}	6.1×10^{-2}	6.1×10^{-2}
5	8.1×10^{-2}	9.6×10^{-2}	0.13	0.13
10	0.16	0.19	0.26	0.26
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	1.6×10^{-2}	1.1×10^{-2}	1.3×10^{-2}	1.2×10^{-2}
5	6.3×10^{-2}	4.2×10^{-2}	5.3×10^{-2}	4.7×10^{-2}
10	0.21	0.14	0.17	0.15
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}=\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	9.6×10^{-3}	4.6×10^{-3}	4.5×10^{-3}	3.6×10^{-3}
5	1.7×10^{-2}	8.5×10^{-3}	8.7×10^{-3}	7.0×10^{-3}
10	3.2×10^{-2}	1.6×10^{-2}	1.6×10^{-2}	1.3×10^{-2}
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}=\mathrm{Re}(\delta^d_{12})^2_{\mathrm{RR}}\rvert}$		
2	8.6×10^{-3}	4.4×10^{-3}	4.4×10^{-3}	3.6×10^{-3}
5	3.2×10^{-2}	1.4×10^{-2}	1.4×10^{-2}	1.1×10^{-2}
10	8.8×10^{-2}	3.4×10^{-2}	3.2×10^{-2}	2.6×10^{-2}

Table 4. Limits on $\mathrm{Re}(\delta^d_{12})_{AB}$ from ΔM_K with gaugino masses of 1000 GeV

M_{sq} [TeV]	No-QCD, VIA	LO-VIA	LO, B_i	NLO, B_i
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}\rvert}$		
2	5.9×10^{-2}	6.9×10^{-2}	9.4×10^{-2}	9.3×10^{-2}
5	9.6×10^{-2}	0.11	0.15	0.15
10	0.17	0.21	0.28	0.28
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	1.4×10^{-2}	9.7×10^{-3}	1.2×10^{-2}	1.1×10^{-2}
5	4.6×10^{-2}	3.0×10^{-2}	3.9×10^{-2}	3.4×10^{-2}
10	0.14	8.8×10^{-2}	0.11	9.8×10^{-2}
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}=\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	4.2×10^{-2}	9.8×10^{-3}	7.8×10^{-3}	6.0×10^{-3}
5	2.2×10^{-2}	1.1×10^{-2}	1.1×10^{-2}	8.5×10^{-3}
10	3.6×10^{-2}	1.8×10^{-2}	1.8×10^{-2}	1.4×10^{-2}
		$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}=\mathrm{Re}(\delta^d_{12})^2_{\mathrm{RR}}\rvert}$		
2	8.0×10^{-3}	4.2×10^{-3}	4.3×10^{-3}	3.5×10^{-3}
5	2.5×10^{-2}	1.2×10^{-2}	1.2×10^{-2}	9.7×10^{-3}
10	6.8×10^{-2}	2.9×10^{-2}	2.9×10^{-2}	2.3×10^{-2}

Table 5. Limits on $\mathrm{Im}(\delta^d_{12})_{AB}$ from ϵ_K with gaugino masses of 250 GeV

M_{sq} [TeV]	No-QCD, VIA	LO-VIA	LO, B_i	NLO, B_i
		$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{\mathrm{LL}}\rvert}$		
2	4.0×10^{-3}	4.7×10^{-3}	6.4×10^{-3}	6.4×10^{-3}
5	9.7×10^{-3}	1.2×10^{-2}	1.6×10^{-2}	1.6×10^{-2}
10	2.0×10^{-2}	2.4×10^{-2}	3.2×10^{-2}	3.2×10^{-2}
		$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	2.7×10^{-3}	1.8×10^{-3}	2.3×10^{-3}	2.1×10^{-3}
5	1.3×10^{-2}	8.4×10^{-3}	1.1×10^{-2}	9.4×10^{-3}
10	4.5×10^{-2}	2.9×10^{-2}	3.7×10^{-2}	3.2×10^{-2}
		$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{\mathrm{LR}}=\mathrm{Im}(\delta^d_{12})^2_{\mathrm{LR}}\rvert}$		
2	8.6×10^{-4}	4.5×10^{-4}	4.6×10^{-4}	3.7×10^{-4}
5	2.0×10^{-3}	1.0×10^{-3}	1.0×10^{-3}	8.3×10^{-4}
10	3.9×10^{-3}	2.0×10^{-3}	2.0×10^{-3}	1.6×10^{-3}
		$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{\mathrm{LL}}=\mathrm{Im}(\delta^d_{12})^2_{\mathrm{RR}}\rvert}$		
2	1.4×10^{-3}	6.7×10^{-4}	6.6×10^{-4}	5.4×10^{-4}
5	5.4×10^{-3}	2.1×10^{-3}	2.0×10^{-3}	1.6×10^{-3}
10	1.4×10^{-2}	4.7×10^{-3}	4.5×10^{-3}	3.6×10^{-3}

We have discussed the residual uncertainty of our results that arise from our ignorance of the NLO-QCD corrections to the matching coefficients.

Our analysis confirms that FCNC suppression is not easily explained by a split sparticle mass spectrum without some amount of fine-tuning. These problems can be solved only if further assumptions about these kinds of models are made, e.g., flavor symmetry or dynamical generation of degenerate scalar masses [16].

In order to perform a complete analysis of SUSY-FCNC effects, chargino contributions should be included. It is also interesting to extend this kind of analysis to $\Delta B = 2$ processes, once the calculation of B parameters for the B–$\overline{\mathrm{B}}$ system parameters (which is in progress [19]) is completed.

Table 6. Limits on $\mathrm{Im}(\delta_{12}^{d})_{AB}$ from ϵ_K with gaugino masses of 500 GeV

M_{sq} [TeV]	No-QCD, VIA	LO-VIA	LO, B_i	NLO, B_i
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LL}}\rvert}$		
2	5.0×10^{-3}	5.9×10^{-3}	8.0×10^{-3}	7.9×10^{-3}
5	1.1×10^{-2}	1.3×10^{-2}	1.7×10^{-2}	1.7×10^{-2}
10	2.1×10^{-2}	2.5×10^{-2}	3.4×10^{-2}	3.3×10^{-2}
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LR}}\rvert}$		
2	2.0×10^{-3}	1.4×10^{-3}	1.8×10^{-3}	1.6×10^{-3}
5	8.3×10^{-3}	5.5×10^{-3}	7.0×10^{-3}	6.2×10^{-3}
10	2.7×10^{-2}	1.8×10^{-2}	2.2×10^{-2}	2.0×10^{-2}
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LR}}=\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LR}}\rvert}$		
2	1.3×10^{-3}	6.0×10^{-4}	5.9×10^{-4}	4.7×10^{-4}
5	2.2×10^{-3}	1.1×10^{-3}	1.1×10^{-3}	9.1×10^{-4}
10	4.2×10^{-3}	2.1×10^{-3}	2.1×10^{-3}	1.7×10^{-3}
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LL}}=\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{RR}}\rvert}$		
2	1.1×10^{-3}	5.8×10^{-4}	5.8×10^{-4}	4.7×10^{-4}
5	4.2×10^{-3}	1.9×10^{-3}	1.8×10^{-3}	1.5×10^{-3}
10	1.1×10^{-2}	4.4×10^{-3}	4.2×10^{-3}	3.4×10^{-3}

Table 7. Limits on $\mathrm{Im}(\delta_{12}^{d})_{AB}$ from ϵ_K with gaugino masses of 1000 GeV

M_{sq} [TeV]	No-QCD, VIA	LO-VIA	LO, B_i	NLO, B_i
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LL}}\rvert}$		
2	7.7×10^{-3}	9.0×10^{-3}	1.2×10^{-2}	1.2×10^{-2}
5	1.3×10^{-2}	1.5×10^{-2}	2.0×10^{-2}	2.0×10^{-2}
10	2.3×10^{-2}	2.7×10^{-2}	3.7×10^{-2}	3.6×10^{-2}
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LR}}\rvert}$		
2	1.8×10^{-3}	1.3×10^{-3}	1.6×10^{-3}	1.4×10^{-3}
5	6.0×10^{-3}	4.0×10^{-3}	5.0×10^{-3}	4.5×10^{-3}
10	1.8×10^{-2}	1.1×10^{-2}	1.5×10^{-2}	1.3×10^{-2}
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LR}}=\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LR}}\rvert}$		
2	5.5×10^{-3}	1.3×10^{-3}	1.0×10^{-3}	7.8×10^{-4}
5	2.9×10^{-3}	1.4×10^{-3}	1.4×10^{-3}	1.1×10^{-3}
10	4.7×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	1.9×10^{-3}
		$\sqrt{\lvert\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{LL}}=\mathrm{Im}(\delta_{12}^{d})^2_{\mathrm{RR}}\rvert}$		
2	1.0×10^{-3}	5.5×10^{-4}	5.6×10^{-4}	4.6×10^{-4}
5	3.3×10^{-3}	1.6×10^{-3}	1.6×10^{-3}	1.3×10^{-3}
10	8.9×10^{-3}	3.8×10^{-3}	3.7×10^{-3}	3.0×10^{-3}

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Appendix

We give here the numerical values for the magic numbers of (9). Only the nonvanishing entries are shown:

$$\begin{aligned} a_{(r)} &= (0.29, -1.1, 0.14, -0.69, 0.79) \\ a'_{(r)} &= (0.46, -1.8, 0.23, -1.1, 1.3) \end{aligned}$$

$$\begin{array}{ll} b_{11}^{(r)} = (1.5, 0, 0, 0, 0) & c_{11}^{(r)} = (-3.4, 0, 0, 0, 0) \\ b_{22}^{(r)} = (0, 0.0048, 1.1, 0, 0) & c_{22}^{(r)} = (0, -0.12, 2.8, 0, 0) \\ b_{23}^{(r)} = (0, -0.0073, 0, 0, 0) & c_{23}^{(r)} = (0, 0.12, 1.2, 0, 0) \\ b_{32}^{(r)} = (0, -0.23, 0.47, 0, 0) & c_{32}^{(r)} = (0, 6.3, 2.0, 0, 0) \\ b_{33}^{(r)} = (0, 0.34, 0, 0, 0) & c_{33}^{(r)} = (0, -6.2, 0.88, 0, 0) \\ b_{44}^{(r)} = (0, 0, 0, 0.52, -0.017) & c_{44}^{(r)} = (0, 0, 0, -5.7, 1.8) \\ b_{45}^{(r)} = (0, 0, 0, 0.99, -2.2) & c_{45}^{(r)} = (0, 0, 0, -26, -27) \\ b_{54}^{(r)} = (0, 0, 0, -0.00051, 0.020) & c_{54}^{(r)} = (0, 0, 0, 0.0086, -0.77) \\ b_{55}^{(r)} = (0, 0, 0, -0.00096, 2.5) & b_{55}^{(r)} = (0, 0, 0, 0.040, 12) \end{array}$$

$$\begin{array}{ll} d_{11}^{(r)} = (1.0, 0, 0, 0, 0) & e_{11}^{(r)} = (-1.5, 0, 0, 0, 0) \\ d_{22}^{(r)} = (0, 0, 1.0, 0, 0) & e_{22}^{(r)} = (0, 0.70, -4.9, 0, 0) \\ d_{23}^{(r)} = (0, 0, 0, 0, 0) & e_{23}^{(r)} = (0, -1.1, 0, 0, 0) \\ d_{32}^{(r)} = (0, -0.67, 0.67, 0, 0) & e_{32}^{(r)} = (0, -27, -9.0, 0, 0) \\ d_{33}^{(r)} = (0, 1.0, 0, 0, 0) & e_{33}^{(r)} = (0, 40, 0, 0, 0) \\ d_{44}^{(r)} = (0, 0, 0, 1.0, -0.015) & e_{44}^{(r)} = (0, 0, 0, 22, 0.46) \\ d_{45}^{(r)} = (0, 0, 0, 1.9, -1.9) & e_{45}^{(r)} = (0, 0, 0, 41, 58) \\ d_{54}^{(r)} = (0, 0, 0, -0.0081, 0.0081) & e_{54}^{(r)} = (0, 0, 0, 0.15, -0.12) \\ d_{55}^{(r)} = (0, 0, 0, -0.015, 1.0) & e_{55}^{(r)} = (0, 0, 0, 0.29, -15) \end{array}$$

$$\begin{aligned} f_{11}^{(r)} &= (1.5, 0, 0, 0, 0) \\ f_{22}^{(r)} &= (0, 0, 4.2, 0, 0) \\ f_{23}^{(r)} &= (0, 0, 1.1, 0, 0) \\ f_{32}^{(r)} &= (0, 33, 2.8, 0, 0) \\ f_{33}^{(r)} &= (0, -41, 0.70, 0, 0) \\ f_{44}^{(r)} &= (0, 0, 0, -23, 0.40) \\ f_{45}^{(r)} &= (0, 0, 0, -72, -28) \\ f_{54}^{(r)} &= (0, 0, 0, 0.18, -0.21) \\ f_{55}^{(r)} &= (0, 0, 0, 0.57, 15) \end{aligned}$$

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